Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 16 January 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

Topic: Factor and remainder theorem

1. $f(x) = ax^3 + 3x^2 - 8x + 2$ where a is a constant

-1

Given that when f(x) is divided by $(x - 2)$ the remainder is 3, find the value of a.

(3)

Background info:
The factor theorem is a special case of the remainder theorem!
The factor theorem is just a remainder of zero, hence we can apply the same method and set our answer equal to 3 instead of zero

$f(x)$ divided by $bx \pm a$ gives remainder $f\left(\mp\frac{a}{b}\right)$. Take note of the opposite sign!

Step 1: set what you're dividing by equal to 0

$x - 2 = 0$

$x = 2$

Step 2: Plug the value found into the function

$f(2) = a(2)^3 + 3(2)^2 - 8(2) + 2$

$f(2) = 8a + 12 - 16 + 2$

$f(2) = 8a - 2$

Step 3: (only do this if solving for an unknown)

set equal to the remainder

$8a - 2 = 3$

$8a = 5$

$\boxed{a = \frac{5}{8}}$

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Question 1 continued

(Total for Question 1 is 3 marks)

2. Find the coefficient of the term in x^7 of the binomial expansion of

$$\left(\frac{3}{8} + 4x\right)^{12}$$

giving your answer in simplest form.

(3)

Formula for binomial expansion:

$$(a+b)^n = {}^nc_0 a^{n-0} b^0 + {}^nc_1 a^{n-1} b^1 + {}^nc_2 a^{n-2} b^2 + \ldots + {}^nc_n a^{n-1} b^1$$

Here we have $\left(\frac{3}{8} + 4x\right)^{12}$

We only want 1 term, so we don't need to find the entire expansion

Each term looks like ${}^{12}c_? \left(\frac{3}{8}\right)^{12-?} (4x)^?$

<u>Way 1</u>: Inspection

$${}^{12}c_? \left(\frac{3}{8}\right)^{12-?} (4x)^?$$

We want the x to have a power of 7 hence $? = 7$

$${}^{12}c_7 \left(\frac{3}{8}\right)^{12-7} (4x)^7$$

$$= {}^{12}c_7 \left(\frac{3}{8}\right)^5 (4x)^7$$

Simplify using indices rule $(ax)^n = a^n x^n$

$$= 792\left(\frac{243}{32768}\right)(16,384x^7)$$

$$= 96,288x^7$$

$\boxed{\text{coefficient} = 96,288}$

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Question 2 continued

Way 2: use algebra - call the power r and solve for it

$$^{12}C_r \left(\frac{3}{8}\right)^{12-r} (4x)^r$$

$$= {}^{12}C_r \left(\frac{3}{8}\right)^{12-r} (4)^r (x)^r$$

We want the x to have a power of 7 hence $r = 7$

replace r with value found

$$= {}^{12}C_7 \left(\frac{3}{8}\right)^{12-7} (4)^7 (x)^7$$

$$= 792\left(\frac{243}{32768}\right)(16,384)x^7$$

$$= 96,228\,x^7$$

$\boxed{\text{coefficient} = 96,288}$

(Total for Question 2 is 3 marks)

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Topic: Circles

3. The circle C

- has centre $A(3, 5)$
- passes through the point $B(8, -7)$

(a) Find an equation for C.

(3)

The points M and N lie on C such that MN is a chord of C.

Given that MN

- lies above the x-axis
- is parallel to the x-axis
- has length $4\sqrt{22}$

(b) find an equation for the line passing through points M and N.

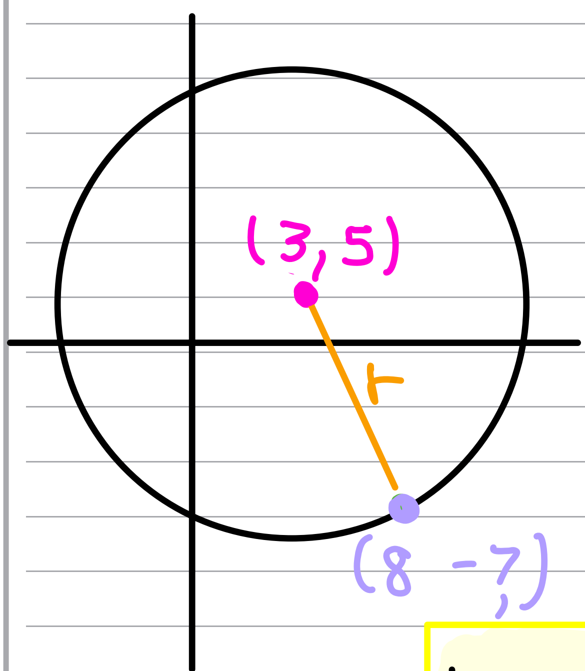
(3)

a)

Equation of a circle

$(x-a)^2+(y-b)^2=r^2$

centre $=(a,b)$ and radius $=r$



Use the distance formula to find r

$r=\sqrt{(8-3)^2+(-7-5)^2}$

$=\sqrt{5^2+(-12)^2}$

$r=13$

$\boxed{(x-3)^2+(y-5)^2=13^2}$

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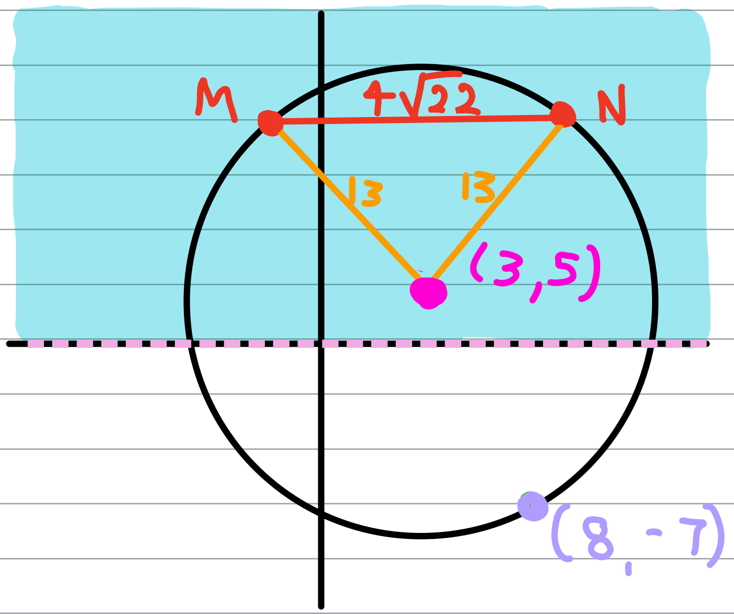
Question 3 continued

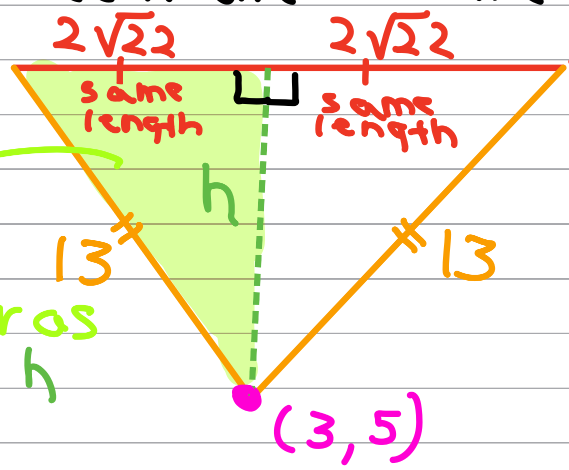
b)



above the x axis

— must be parallel to this line

Let's concentrate on the triangle:



This red line has the equation y = number

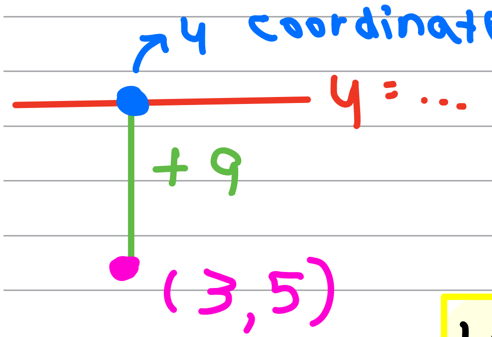
Use Pythagoras to find h

Pythagoras: $h^2 + (2\sqrt{22})^2 = 13^2$

$h^2 = 169 - 88$

$h = 9$

Use symmetry now



hence $y = 5 + 9$

$\boxed{y = 14}$

(Total for Question 3 is 6 marks)

Topic: Exponential graphs & trapezium rule

4. (a) Sketch the curve with equation

$$y = a^{-x} + 4$$

where a is a constant and $a > 1$

On your sketch show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote to the curve.

(3)

x	-4	-1.5	1	3.5	6	8.5
y	13	6.280	4.577	4.146	4.037	4.009
	y_0	y_1	y_2	y_3	y_4	y_5

The table above shows corresponding values of x and y for $y = 3^{-\frac{1}{2}x} + 4$

The values of y are given to four significant figures, as appropriate.

Using the trapezium rule with all the values of y in the table,

(b) find an approximate value for

$$\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) \mathrm{d}x$$

giving your answer to two significant figures.

(3)

(c) **Using the answer to part (b)**, find an approximate value for

(i) $\displaystyle\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x}\right) \mathrm{d}x$

(ii) $\displaystyle\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) \mathrm{d}x + \int_{-8.5}^{4} \left(3^{\frac{1}{2}x} + 4\right) \mathrm{d}x$

(3)

a) $y = a^{-x} + 4$

We need to find the important features of this graph which are x intercept, y intercept and the y asymptote

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Question 4 continued

x intercept
let $x=0$:
$y = a^{-0} + 4$
$y = 5$

$(0, 5)$

y intercept:
let $y=0$
$1 = a^{-x} + 4$
$-3 = a^{-x}$
$a^{-x} = -3$
no solution since can't log a negative number hence no y intercept

y asymptote:
$y = a^{-x} + 4$

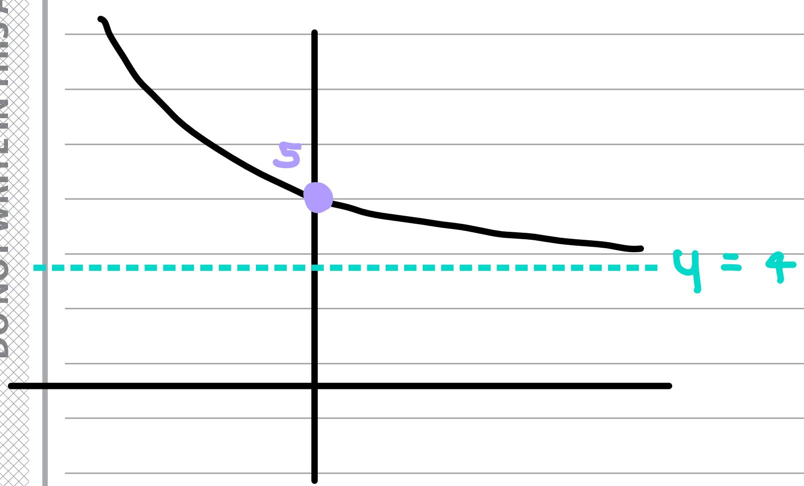
it is the number we + or − from the exponential

Why? a^{-x} can never be 0 since $a^0 \neq 0$. It is always a positive number

An exponential graph has the shape like ◡ or ◟ or ◜ or ◝

We know that we cross the y intercept at $(0, 5)$ and cannot touch the asymptote $y = 4$

$a > 0$ means graph looks like ◡ or ◟



$y = a^{-x} + 4$

reflection in y axis

More advanced reasoning:
- as $x \to \infty$: $e^{-\text{large}} = \frac{1}{e^{\text{large}}} \to 0$
  hence $y \to 4$
- as $x \to -\infty$: $e^{\text{large}} \to \infty$
  hence $y \to \infty$

Question 4 continued

b)

trapezium rule formula:

$$\int_a^b y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + \dots + y_{n-1}) + y_n\right]$$

first y ... last y

where $h = \dfrac{b-a}{\text{number of strips}}$

Note: number of strips = number of coords − 1

Here we get:

$$\frac{8.5-4}{5}\left[13 + 2(6.280 + 4.577 + 4.146 + 4.037) + 4.009\right]$$

to find this:
The table gives 6 x coordinates
hence 5 strips

1 2 3 4 5

$= 68.86125\ldots \approx \boxed{89}$

c)

from b) we found $\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) dx = 89$

missing

i) $\int_{-4}^{8.5} 3^{-\frac{1}{2}x}\,dx$

We can see there is no $+4$ in the integral, hence we need to find $\int_{-4}^{8.5} 4\,dx$ and take it away

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Question 4 continued

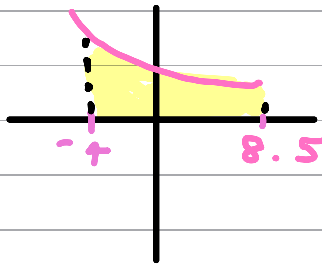
$$\int_{-4}^{8.5} 4 \, dx = \left[4x\right]_{-4}^{8.5} = 4(8.5) - 4(-4)$$

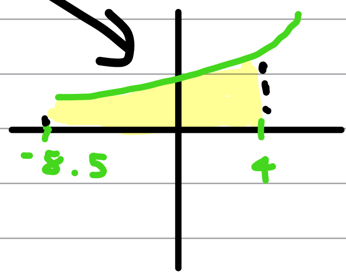
$$= 50$$

$$\int_{-4}^{8.5} 3^{-\frac{1}{2}x} = 69 - 50 = 19$$

ii) $\int_{-4}^{8.5} (3^{-\frac{1}{2}x} + 4)\, dx + \int_{-8.5}^{4} (3^{\frac{1}{2}x} + 4)\, dx$

the power is not negative this time





Notice how the areas are the same

Note: This is the reason part a) wanted us to draw a graph

$= 69 + 69$

$= \boxed{138}$

(Total for Question 4 is 9 marks)

Topic: Geometric & recursive sequences

5. (i) Find the value of

end

sum $\sum_{r=1}^{\infty} 6 \times (0.25)^r$

starting value

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3 \quad \text{starting point}$$

$$u_{n+1} = \frac{u_n - 3}{u_n - 2} \qquad n \in \mathbb{N}$$

(a) Show that this sequence is periodic.

repeats every certain number of terms

(2)

(b) State the order of this sequence.

how often repeats

(1)

(c) Hence find

$$\sum_{n=1}^{70} u_n$$

(2)

i) $\sum_{r=1}^{\infty} 6(0.25)^r$

$\sum_{r=1}^{\infty}$ means start the sequence at $r = 1$ and sum the terms all the way to infinity

hint: sum to infinity involved a (geometric sequence)

Let's write out some terms

$\underbrace{6(0.25)^1}_{\text{first term}} + 6(0.25)^2 + 6(0.25)^3 + 6(0.25)^4 + \ldots$

This is geometric (multiply by 0.25 each time

- a = first term = 6(0.25)
- r = common ratio = $\dfrac{\text{any term}}{\text{its previous term}}$ = 0.25

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Question 5 continued

Formula for sum to infinity of a geometric series:

$S_\infty = \frac{a}{1-r}$ where a = first term, r = common ratio

$S_\infty = \frac{6(0.25)}{1-0.25} = \frac{1.5}{1-0.25} = 2$

ii) This is a recursive sequence

a) $U_{n+1} = \frac{U_n - 3}{U_n - 2}$ Notice how $n+1$ is 1 more than n

Let's find some terms

We are given $U_1 = 3$ as a starting point

$U_2 = \frac{U_1 - 3}{U_1 - 2} = \frac{3-3}{3-2} = \frac{0}{1} = 0$

$U_3 = \frac{U_2 - 3}{U_2 - 2} = \frac{0-3}{0-2} = \frac{3}{2}$

$U_4 = \frac{U_3 - 3}{U_3 - 2} = \frac{\frac{3}{2} - 3}{\frac{3}{2} - 2} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$

back where started at $U_1 = 3$

Hence we have the sequence:

$\underbrace{3, 0, \frac{3}{2}}_{\text{repeats every 3 terms}}, 3, 0, \frac{3}{2}$

$U_4 = U_1$ hence sequence repeats

Question 5 continued

b) the sequence repeats every 3 terms hence order 3

c) $\sum_{n=1}^{70} U_n = U_1 + U_2 + U_3 + \dots + U_{70}$

Using our results from in part a) we get

$$= \underbrace{3 + 0 + \frac{3}{2}}_{} + 3 + 0 + \frac{3}{2} + \dots$$

Sum of 3 terms $= 3 + 0 + \frac{3}{2} = \frac{9}{2}$

We have 70 terms. How many lots of 3 fit in?

$\frac{70}{3} = 23$ remainder 1 (since $23 \times 3 = 69$ with 1 left over)

the next term

$$23\left(\frac{9}{2}\right) + 3$$

$= 106.5$

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Question 5 continued

(Total for Question 5 is 8 marks)

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Topic: logs

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that –1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

a)

1 log only

Goal: To get the form $\log_{\square} \square = \text{number}$ and use log rule ③ to get rid of the log in order to solve for x

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

Firstly let's get the logs on one side

$$2\log_4(x+3) + \log_4 x - \log_4(4x+2) = \frac{1}{2}$$

needs to be a 1

Log rule formulae:

$a\log_b c \Leftrightarrow \log_b c^a$

"We can bring the power up or down"

$$\log_4(x+3)^2 + \log_4 x - \log_4(4x+2) = \frac{1}{2}$$

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Question 6 continued

Log rule formula:

$1\log_c a + 1\log_c b \Leftrightarrow \log_c a\times b$

$1\log_c a - 1\log_c b \Leftrightarrow \log_c \frac{a}{b}$

"two logs added/subtracted go to one log multiplied/divided and vice versa"

Notice: The same base

The must be a 1 infront of the logs (use rule ① if we don't have this)

$\log_4 \frac{(x+3)^2 \times x}{4x+2} = \frac{1}{2}$

Log rule formula:

$\log_a b = c \Leftrightarrow a^c = b$

"we can turn a log into a power/get rid of the log or vice versa"

we mostly use this rule when solving

$4^{\frac{1}{2}} = \frac{(x+3)^2 x}{4x+2}$

Now use algebra to re-arrange

$2 = \frac{(x^2+6x+9)x}{4x+2}$

multiply

$2(4x+2) = x^3 + 6x^2 + 9x$

$8x+4 = x^3 + 6x^2 + 9x$

$\boxed{x^3 + 6x^2 + x - 4 = 0}$

Question 6 continued

b) -1 is a root means $x - -1$ i.e $x+1$ is a factor (remainder is 0)

There are 3 ways to do this question

Way 1: Polynomial division

Way 2: comparing coefficients

Way 3: synthetic division

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Question 6 continued

(Total for Question 6 is 8 marks)

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model *A* assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model *A*, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.
(3)

Model *B* assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model *B*, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.
(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.
(3)

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Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

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Topic: Proof

8. (i) Use a counter example to show that the following statement is **false**

"$n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

"$n^2 - 2$ is **not** a multiple of 4"

(4)

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Question 8 continued

(Total for Question 8 is 6 marks)

Topic: Trig Solving & Graphs

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

(ii)

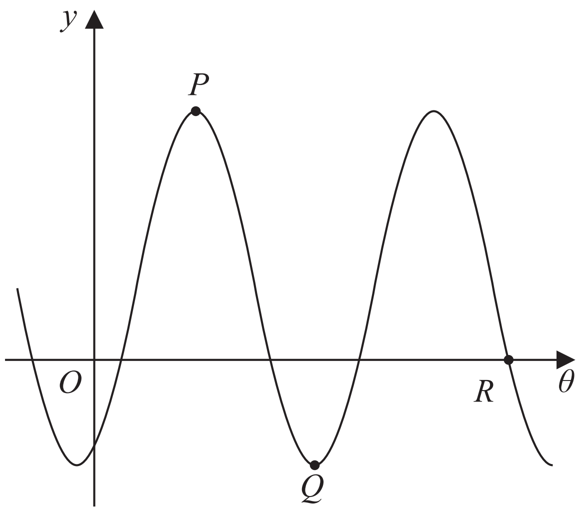

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 14 marks)

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

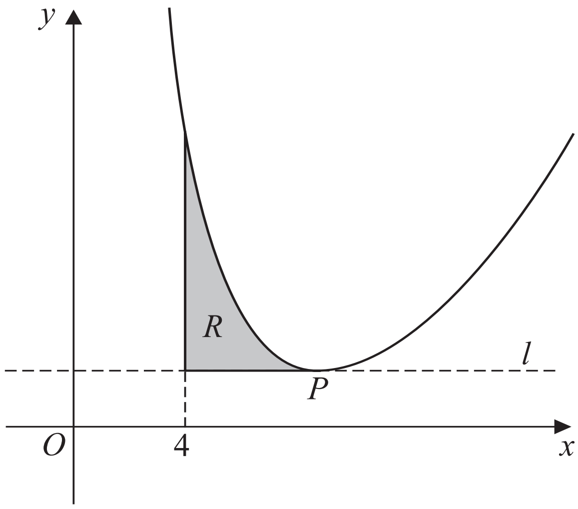

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS